Large Deviation Theory for the Analysis of Power Tansmission Systems Subject to Stochastic Forcing

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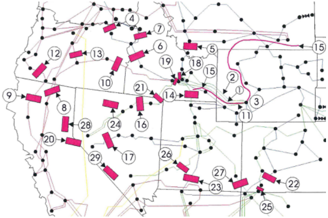
Quantifying the risk of cascading power transmission outages is critical

- Critical for safe planning and operation of the grid
- The growing complexity of the grid render the challenge and importance of this problem more pronounced



Event sequence of the WSCC July 2 & 3 1996 system disturbance [2]

Challenges

- Component outages don't propagate locally along the grid topology
- Necessary to resolve the complex interactions between components
  - Grid dynamics
  - AC power flow
- Rare events: Lack of data to guide data-driven statistical models

Our goal: A generative probabilistic model for cascading failure

Approach: Construct...

1. Analytic, tractable models for probabilities of *individual* component failures
   - Accounting for grid dynamics and AC power flow
   - ...and Load and generation fluctuations
2. *Aggregate* failure model based on individual probabilities

Opportunities

- Stochastic dynamical systems
- Large deviation theory

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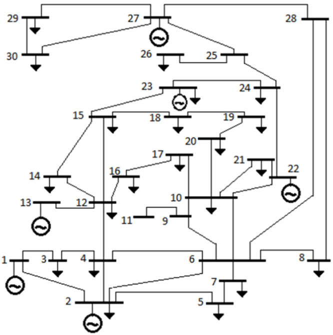
Outline

1. Power transmission network model
2. Individual line failure model
3. Aggregate line failure model

Power transmission network model

Undirected graph $(\mathcal{B}, \mathcal{E})$, with $\mathcal{E}$ the set of transmission lines and $\mathcal{B} \equiv \mathcal{G}$ (generator) $\cup \mathcal{L}$ (load) $\cup \mathcal{S}$ (slack/ref.) the set of nodes/buses



IEEE 30-bus system [3]

Assumptions

- Swing equations to model generation synchronization
- Lossless AC power flow equations
- Frequency-dependent active load

y: Operating conditions

Power transmission network model

DAE dynamics

$$\dot{\theta}_i = \omega_i - \omega_{\mathcal{S}}, \qquad i \in \mathcal{G}$$

$$\dot{\omega}_i = P_i^y - F_i^y(\theta, V) - D_i(\omega_i - \omega_{\mathcal{S}}), \qquad i \in \mathcal{G} \cup \mathcal{S}$$

- Swing equations

Power transmission network model

DAE dynamics

$$
\begin{aligned}
\dot{\theta}_i &= \omega_i - \omega_{\mathcal{S}}, && i \in \mathcal{G} \\
\dot{\omega}_i &= P_i^y - F_i^y(\theta, V) - D_i(\omega_i - \omega_{\mathcal{S}}), && i \in \mathcal{G} \cup \mathcal{S} \\
0 &= P_i^y - F_i^y(\theta, V), && i \in \mathcal{L} \\
0 &= Q_i^y - G_i^y(\theta, V), && i \in \mathcal{L}
\end{aligned}
$$

- Swing equations
- Lossless AC power flow

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Power transmission network model

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-D_{\mathcal{L}}\dot{\theta}_i &= P_i^y - F_i^y(\theta, V), && i \in \mathcal{L}
\end{aligned}
$$

- Swing equations
- Lossless AC power flow
- Load model

Power transmission network model

DAE dynamics

$$\begin{aligned}
\dot{\theta}_i &= \omega_i - \omega_{\mathcal{S}}, & i \in \mathcal{G} \\
\dot{\omega}_i &= P_i^y - F_i^y(\theta, V) - D_i(\omega_i - \omega_{\mathcal{S}}), & i \in \mathcal{G} \cup \mathcal{S} \\
0 &= P_i^y - F_i^y(\theta, V), & i \in \mathcal{L} \\
0 &= Q_i^y - G_i^y(\theta, V), & i \in \mathcal{L} \\
-D_{\mathcal{L}}\dot{\theta}_i &= P_i^y - F_i^y(\theta, V), & i \in \mathcal{L}
\end{aligned}$$

Singularly-perturbed ODE system

$$\dot{x} = \begin{bmatrix} \dot{\omega}_{\mathcal{G}\cup\mathcal{S}} \\ \dot{\theta}_{\mathcal{G}\cup\mathcal{L}} \\ \dot{V}_{\mathcal{L}} \end{bmatrix} = \begin{bmatrix} -M_{\mathcal{G}}^{-1} D_{\mathcal{G}} M_{\mathcal{G}}^{-1} & -M_{\mathcal{G}}^{-1} T_1^\top & 0 \\ T_1 M_{\mathcal{G}}^{-1} & -T_2 D_{\mathcal{L}}^{-1} T_2^\top & 0 \\ 0 & 0 & D_V^{-1} \mathcal{I}_{\mathcal{L}} \end{bmatrix} \nabla \mathcal{H}^y(x)$$

$x \in \mathbb{R}^d$, with “energy” function

$$\mathcal{H}^y(x) = \frac{1}{2}\omega_{\mathcal{G}\cup\mathcal{S}}^\top M_{\mathcal{G}}\, \omega_{\mathcal{G}\cup\mathcal{S}} + \frac{1}{2} v_{\mathcal{L}}^{\mathsf{H}} B^y\, v_{\mathcal{L}} + \left(P_{\mathcal{G}\cup\mathcal{L}}^y\right)^\top \theta_{\mathcal{G}\cup\mathcal{L}} + \left(Q_{\mathcal{L}}^y\right)^\top \log V_{\mathcal{L}}$$

Port-Hamiltonian form

The singularly-perturbed model is of Port-Hamiltonian form

$$\dot{x} = (J - S)\nabla\mathcal{H}^y(x)$$

where J is skew-symmetric, and $S \succeq 0$

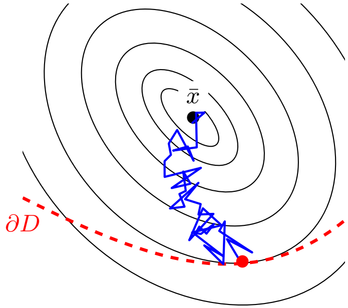
Stochastic model [4]

To account for noise in generation and load we introduce white noise:

$$\mathrm{d}x_t^\tau = (J - S)\nabla\mathcal{H}^y\left(x_t^\tau\right)\mathrm{d}t + \sqrt{2\tau}S^{1/2}\mathrm{d}W_t$$

where τ is the noise strength/“temperature”, and $W_t \in \mathbb{R}^d$ is a vector of d independent Weiner processes

Modeling line failures



- Line energy constraint $\Theta_l(x_t) < \Theta_l^{\max}$
- Line fails if dynamics exit the basin of attraction around $\bar{x}$ across ∂D

$$D \equiv \{x \colon \Theta_l(x) < \Theta_l^{\max}\}$$

- **Goal:** Estimate distribution of first exit times

$$T_{\partial D}^{\tau} \equiv \inf\{t > 0, x_t^{\tau} \in \partial D\}$$

- In general, $\langle b(x), n(x) \rangle < 0$ (*non-characteristic*, $n(x)$: outward unit vector normal to ∂D), so we can employ the large deviation theory for escapes across non-characteristic surfaces

Freidlin-Wentzell large deviation theory

For the subdomain $D \subset \mathbb{R}^d$ with non-characteristic surface ∂D,

$$\lim_{\tau \to 0} \tau \log \mathbb{E} T^{\tau}_{\partial D} = \min_{x \in \partial D} V(\bar{x}, x)$$

with *quasipotential*

$$V(\bar{x}, x) \equiv \inf \left\{ S^{\bar{x}}_{[0,T]}(\phi_t) \colon \phi_t(0) = \bar{x},\ \phi_t(T) = x,\ T > 0 \right\}$$

$$S^{\bar{x}}_{[0,T]}(\phi_t) = \frac{1}{4} \int_0^T \left\langle \left[\dot{\phi}_t - b(\phi_t) \right], \left[\sigma(\phi_t) \sigma(\phi_t)^\top \right]^+ \left[\dot{\phi}_t - b(\phi_t) \right] \right\rangle \mathrm{d}t$$

Transverse decomposition

There are smooth functions $U \colon D \cup \partial D \to \mathbb{R}^d$, $l \colon D \cup \partial D \to \mathbb{R}^d$ such that

- $b(x) = -\sigma(x)\sigma(x)^\top \nabla U(x) + l(x)$
- $\langle \nabla U(x), l(x) \rangle = 0$

Assuming this decomposition, we have $V(\bar{x}, x) = U(x) - U(\bar{x})$

Freidlin-Wentzell large deviation theory

During the *quasi-stationary phase*

$$1 \ll t \ll \exp\left[\min_{x\in\partial D} \frac{U(x)-U(\bar{x})}{\tau}\right],\text{ we have}$$

$$\frac{d}{dt}\mathbb{P}\left[T^{\tau}_{\partial D} > t\right] \approx -\int_{\partial D} \langle j^{\tau}(x), n(x)\rangle \,\mathrm{d}S(x) \equiv -\lambda^{\tau}$$

- λ^{τ}: (quasi-stationary) Exit rate
- j^{τ}: (quasi-stationary) Probability current

For $\operatorname{div} l(x) = 0$,

$$j^{\tau}(x) = \sqrt{\frac{\det \operatorname{Hess} U(\bar{x})}{(2\pi\tau)^d}} \exp\left(-\frac{U(x)-U(\bar{x})}{\tau}\right) \Big\langle \sigma(x)\sigma(x)^{\top} U(x) + l(x), n(x)\Big\rangle$$

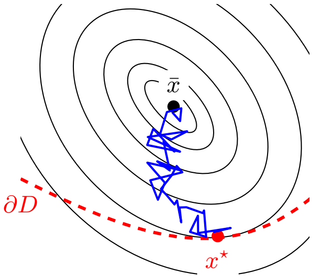
(Bouchet-Reygner [1])

Asymptotic exit rate

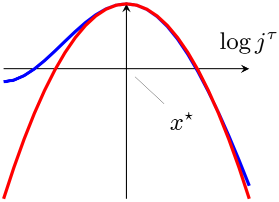
Our model has a transverse decomposition with $U(x) = \mathcal{H}^y(x)$, $l(x) = J\nabla\mathcal{H}^y(x)$, and $\sigma = S^{1/2}$

- For $\tau \to 0$, the probability current is peaked around

$$x^\star \equiv \underset{x\in\partial D}{\arg\min}\, V(\bar{x}, x) = \underset{\Theta_l(x)=\Theta_l^{\max}}{\arg\min}\, \mathcal{H}^y(x)$$



$x^\star$: Exit point for $\tau \to 0$



Asymptotic exit rate

Laplace surface integral leads to

$$\lambda^\tau \underset{\tau\to 0}{\sim} \nabla^\top \mathcal{H}(x_\star) S \nabla \mathcal{H}(x_\star) \sqrt{\frac{|\det \operatorname{Hess} \mathcal{H}(\bar{x})|}{2\pi\tau B^\star}} \exp\left(-\frac{\mathcal{H}(x_\star) - \mathcal{H}(\bar{x})}{\tau}\right)$$

with $\mathcal{H} \equiv \mathcal{H}^y$, where $B^\star$ is a factor accounting for the curvature of ∂D around the exit point $x^\star$:

$$B^\star \equiv \nabla_x \mathcal{H}(x^\star)^\top L^{-1} \nabla_x \mathcal{H}(x^\star) \det L, \quad L = \operatorname{Hess} \mathcal{H}(x_\star) - k \operatorname{Hess} \Theta_l(x_\star)$$

and k is the Lagrange multiplier of the Θ_l constraint

Individual line failure model

Energy minimizers

$$\bar{x} \equiv \operatorname*{arg\,min}_{\Theta_l(x)<\Theta_l^{\max}} \mathcal{H}^y(x), \quad x^\star \equiv \operatorname*{arg\,min}_{\Theta_l(x)=\Theta_l^{\max}} \mathcal{H}^y(x)$$

Failure rate

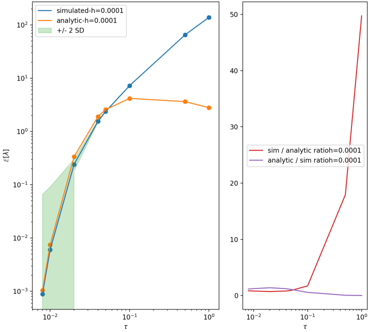
$$\lambda^\tau \underset{\tau\to 0}{\sim} \nabla^\top \mathcal{H}(x_\star) S \nabla \mathcal{H}(x_\star) \sqrt{\frac{|\det \operatorname{Hess} \mathcal{H}(\bar{x})|}{2\pi\tau B^\star}} \exp\left(-\frac{\mathcal{H}(x_\star) - \mathcal{H}(\bar{x})}{\tau}\right)$$

Assumptions

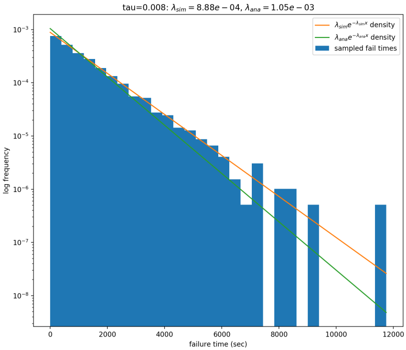
- Non-characteristic transition surface $\partial D = \{x \colon \Theta_l(x) = \Theta_l^{\max}\}$
- $\langle n(x), Sn(x)\rangle > 0$, so not applicable to generator-generator and slack-generator lines

Failure rate validation

3-bus system



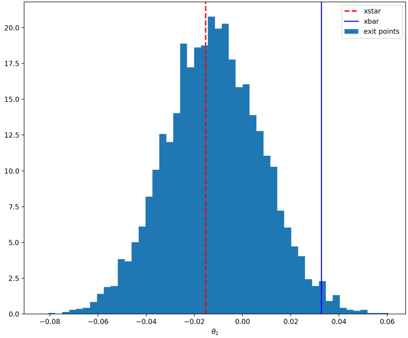
Escape rate vs. τ



Exit time histogram

Line 2 (Generator-load)

Failure rate validation
3-bus system

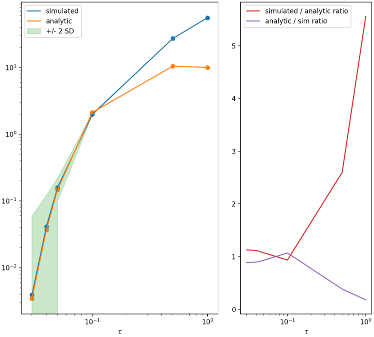


Exit point histogram

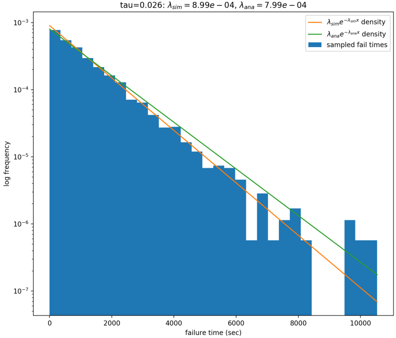
Line 2 (Generator-load)

Failure rate validation

30-bus system



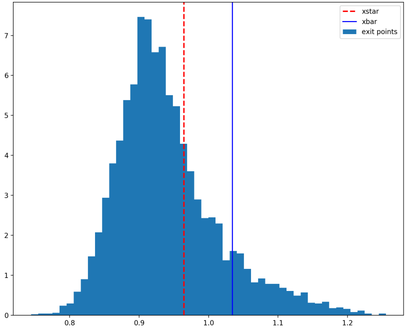
Escape rate vs. τ



Exit time histogram

Line 5 (Slack-load)

Failure rate validation
30-bus system



Exit point histogram

Line 5 (Slack-load)

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Aggregate line failure model

- Event-based discretization of dynamics
- Simulate cascade by jumping between line failures with probability given by the individual line failure rates
- Line failure sequence S and its permutations $\sigma(S)$ produce the same $\bar{x}$ and λ_l^τ

Algorithm Kinetic Monte Carlo

Require: Initialize sequence $S \leftarrow \{\varnothing\}$

repeat

  Compute $\bar{x}$ for S

  Compute $x_l^\star$ and λ_l^τ for each line l given S

  Compute aggregate rate $\lambda_{S\to\hat{S}} = \sum_l \lambda_{S\to S\cup l}$

  Sample failure time as $\Delta t \sim \mathrm{Exp}\left(\lambda_{S\to S\cup l}\right)$

  Sample failed line $\hat{l}$ according to its contribution to the aggregate rate

  $t \leftarrow t + \Delta t$

  $S \leftarrow \hat{S} \equiv S \cup \hat{l}$

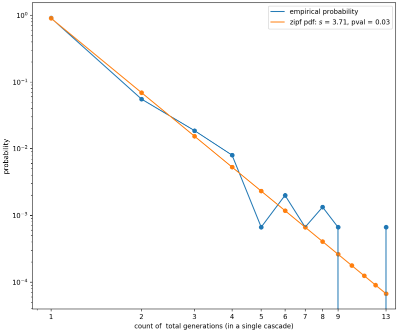
until End cascade

Aggregate line failure model

- Split simulated cascade into “generations” (sequences of failures in 1 min timeframe)
- Observed power-law (Zipf) distribution of count of generations in a cascade

Aggregate line failure model

- Split simulated cascade into "generations" (sequences of failures in 1 min timeframe)
- Observed power-law (Zipf) distribution of count of generations in a cascade
- KMC model resolves power-law distribution



Empirical distribution of counted total generations for cascade of 118-bus system

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Conclusions

A generative probabilistic model for quantifying risk of cascading failure

- Formulated a stochastic Port-Hamiltonian model of transmission network dynamics subject to stochastic forcing
- Individual line failure model: Large deviation theory employed to evaluate failure rates of each line
- Aggregate line failure model: KMC algorithm based on individual line failure rates

References

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[4] C. Matthews, B. Stadie, J. Weare, M. Anitescu, and C. Demarco. Simulating the stochastic dynamics and cascade failure of power networks. *arXiv e-prints*, art. arXiv:1806.02420, Jun 2018.